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ARTICLE



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# The impact of stochastic lead times on the bullwhip effect – a theoretical insight

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## ABSTRACT

In this article, we analyze the models quantifying the bullwhip effect in supply chains with stochastic lead times and find advantages and disadvantages of their approaches to the bullwhip problem. Moreover, using computer simulation, we find interesting insights into the bullwhip behavior for a particular instance of a multi-echelon supply chain with constant customer demands and random lead times. We confirm the recent finding of Michna and Nielsen that under certain circumstances lead time signal processing is by itself a fundamental cause of bullwhip effect just like demand-signal processing is. The simulation also shows that in this supply chain the delay parameter of demand forecasting smooths the bullwhip effect at the manufacturer level much faster than the delay parameter of lead time forecasting. Additionally, in the supply chain with random demands, the reverse behavior is observed, that is, the delay parameter of lead time forecasting smooths bullwhip effect at the retailer stage much faster than the delay parameter of demand forecasting. At the manufacturer level, the delay parameter of demand forecasting and the delay parameter of lead time forecasting dampen the effect with a similar strength.

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Supply chain; Bullwhip effect; lead time demand; lead time forecasting; stochastic lead time

## 1. Introduction

In this paper, we investigate and review the problem of stochastic lead times in supply chains. Our goal is to present analytical models in which bullwhip effect is quantified and especially stochastic lead times are included. Based on theoretical models, we find the parameters responsible for the bullwhip effect and quantify their impact on bullwhip. Moreover, we present simulation experiments where we confirm theoretical results of the presented models and draw conclusions about more complicated and realistic supply chains for which theoretical models do not exist. Recently, the impact of stochastic lead times on the bullwhip effect has been studied very intensively but lead time forecasting and its influence on the bullwhip effect requires both deep analysis and new models. First, we analyze a model (Michna, Nielsen, & Nielsen, 2013) (a slight modification of Kim, Chatfield, Harrison, & Hayya, 2006) where stochastic lead times and lead time demand forecasting are considered. In this model, the analytical expression for the bullwhip effect

measure indicates that the distribution of lead times (the probability of the longest lead time, its expectation, and variance) and the delay parameter of the lead time demand prediction are the main causes of the bullwhip phenomenon. Next, we investigate the work of Duc, Luong, and Kim (2008), where stochastic lead times are considered without forecasting and finally a recent model of Michna and Nielsen (2013) where the bullwhip effect is quantified in the presence of lead time forecasting. Moreover, we extend the model of Michna and Nielsen (2013) for more echelons, and – using simulation – we find some interesting insights into the bullwhip effect's behavior when demands and lead times are forecasted separately. We conduct two experiments where we confirm the impact of lead time signal processing on the bullwhip effect in multi-echelons supply chains even if the customer demand is deterministic. Moreover, from the simulations, we draw interesting conclusions about the dampening of bullwhip throughout the increase of the number of past observations included in the demand and lead time forecasting.

The remainder of the paper is structured as follows: The next section analyses the current main models of supply chains with stochastic lead times, which quantify bullwhip effect. Moreover, we expand and modify some of the results and add computer simulation of multi-echelon supply chains. Finally, we present conclusions and future research opportunities.

## 2. Models with stochastic lead times

Nielsen, Michna, and Nielsen (2017) establish that at least in some cases, lead times can be considered to be independent and identically distributed (i.i.d.). Thus, a logical next step is to analyze the current state of research into supply chains where lead times are assumed to be stochastic. Lead times are typically regarded as the second main cause of the bullwhip effect after demand forecasting (see, e.g. Chen, Drezner, Ryan, & Simchi-Levi, 2000a). Theoretically, zero lead time would eliminate the bullwhip effect. In practice, lead times consist of two components: physical delays and information delays. In models, one does not distinguish between these components as lead time is the time between when a member of a supply chain places an order and the epoch when the product is delivered to the member. The assumption that the lead time is constant is only theoretical. Undoubtedly, in many supply chains, physical and information delays vary, which means that a member of a supply chain does not know the values of the future lead times and must predict them using past observed lead times. This is supported by, for example, Disney, Maltz, Wang, and Warburton Roger (2016) and Nielsen et al. (2017).

The main difference in supply chain models with stochastic lead times lies in the definition of the lead time demand forecast. Let us recall that the following defines the lead time demand at the beginning of a period  $t$  (at a certain stage of the supply chain):

$$D_t^L = D_t + D_{t+1} + \dots + D_{t+L_t-1} = \sum_{i=0}^{L_t-1} D_{t+i}, \quad (1)$$

where  $D_t, D_{t+1}, \dots$  denote demands (from a stage below) during  $t, t+1, \dots$  periods, and  $L_t$  is the lead time of the order placed at the beginning of the period  $t$  (order placed to a stage above). This value sets down the demand during a lead time. The demands come from the stage right below, and the lead times come from the supplier right above. That is,

they are delivery lead times of the supplier that is right above the receiving supply chain member. This quantity is necessary to place an order. The member of the supply chain does not know its value at time  $t$  but must predict it to place an order. Thus, to analyze the bullwhip effect, we need to examine the definitions of lead time demand forecasting  $\hat{D}_t^L$ . The approaches to this problem vary greatly in models with stochastic lead times, and some of them cannot be feasible in practice. The problem of the lead time demand prediction is much more complicated if lead times are stochastic as the forecasting of lead time demand is then much more cumbersome. In all the presented models, we consider a simple two-stage supply chain consisting of customers, a retailer, and a manufacturer. Moreover, we will assume that the retailer uses the order-up-to-level policy (which is optimal in the sense that it minimizes the total discounted linear holding and backorder costs if there are no crossovers), then the level of the inventory at time  $t$  has to be:

$$S_t = \hat{D}_t^L + z\hat{\sigma}_t, \quad (2)$$

where  $\hat{D}_t^L$  is the lead time demand forecast at the beginning of the period  $t$  (i.e. the prediction of the quantity given in (1)), and:

$$\hat{\sigma}_t^2 = \text{Var}(D_t^L - \hat{D}_t^L), \quad (3)$$

is the variance of the forecast error for the lead time demand, and  $z$  is the normal  $z$ -score that specifies the probability that demand is fulfilled by the on-hand inventory, and it can be found based on a given service level. The definition of  $\hat{\sigma}_t^2$  differs in articles (see, e.g. Chen et al., 2000a; Chen, Ryan, & Simchi-Levi, 2000b; Duc et al., 2008; Kim et al., 2006) which results in slightly different formulas of the bullwhip effect measure (e.g. equality instead of inequality). Practically, instead of variance, one must use the empirical variance of  $D_t^L - \hat{D}_t^L$ . This complicates the theoretical derivation of the bullwhip measure significantly. Moreover, we should note that the estimation of  $\hat{\sigma}_t^2$  increases the size of the bullwhip. Under the above assumptions, using the order-up-to-level policy quantity  $q_t$  placed by the retailer at the beginning of a period  $t$  is:

$$q_t = S_t - S_{t-1} + D_{t-1}, \quad (4)$$

Negative values of  $q_t$  are allowed; they correspond to returns.

## 2.1. Lead time demand forecasting using moving average

We will analyze a model which is a slight modification of Kim et al. (2006). In this model, the bullwhip effect is quantified in the presence of stochastic lead times where lead time demand is forecast using the moving-average method (see Michna et al., 2013 for a deeper discussion of Kim et al., 2006 and proofs). We assume that the customer's demands constitute an i.i.d.  $\{D_t\}_{t=-\infty}^{\infty}$ , and the lead times  $\{L_t\}_{t=-\infty}^{\infty}$  are also independent and identically distributed, and the sequences are mutually independent. Let us put  $\mathbb{E}D_t = \mu_D$ ,  $\text{Var}D_t = \sigma_D^2$ ,  $\mathbb{E}L_t = \mu_L$  and  $\text{Var}L_t = \sigma_L^2$ . Additionally, we assume that lead times are bounded random variables, that is,  $L_i \leq M$ , where  $M$  is a positive integer. This assumption is not adopted in Kim et al. (2006), but it is necessary to make the prediction of lead time demands. More precisely, we get back at least  $M$  periods to forecast lead time demand, that is, at time  $t$ , we surely know lead time demands of times

$t-M, t-M-1, \dots$ , and we may not know lead time demands of times  $t-M+1, t-M+2, \dots$ . The orders with these lead times may not be realized at time  $t$ , or if they were realized, they would be improbably short, and that means that including these lead times could distort the forecasting. We will need to know the distribution of  $L_t$  to calculate the bullwhip effect measure, that is, we assume that:

$$\mathbb{P}(L_t = k) = p_k,$$

where  $k = 1, 2, \dots, M$  and  $k$  are the number of periods (in practice, we estimate these probabilities). Thus, the prediction of the lead time demand at time  $t$  using the method of moving average with the length  $n$  is as follows.

$$\hat{D}_t^L = \frac{1}{n} \sum_{j=0}^{n-1} D_{t-M-j}^L. \quad (5)$$

**Theorem 1** *Under the above assumptions and for  $n \geq M$ , the bullwhip effect measure is as follows:*

$$BM = \frac{\text{Var}q_t}{\text{Var}D_t} = 1 + \frac{2p_M}{n} + \frac{2\mu_L}{n^2} + \frac{2\mu_D^2\sigma_L^2}{\sigma_D^2n^2} \quad (6)$$

**Proof:** See Michna et al. (2013).

It is much more difficult to find the bullwhip effect measure under the above assumptions in the case  $n < M$ . In practice, the case  $n \geq M$  is more useful because a large value of  $n$  is much more common in real supply chains. The term  $((2p_M)/n)$  in the formula (6) is the largest one as a function of  $n$ . Thus, the probability of achieving the longest lead time is a crucial parameter in reducing bullwhip. Moreover, if  $p_M = 0$ , and we still get back  $M$  periods in the prediction of lead time demands, then the bullwhip effect measure is reduced by the term  $(O(1/n))$  and is of the form:

$$\frac{\text{Var}q_t}{\text{Var}D_t} = 1 + \frac{2\mu_L}{n^2} + \frac{2\mu_D^2\sigma_L^2}{\sigma_D^2n^2}.$$

Moreover, the formula (6) reveals that the bullwhip effect depends on the lead time distribution through its mean and variance and the probability of the longest lead time. Another important parameter as (6) shows is that the delay parameter  $n$  is responsible for forecasting and carries the information on past lead time demands. It is easy to notice that the delay parameter  $n$  can diminish the bullwhip effect if it increases, which practically means that the more accurate forecasting, the smaller the bullwhip effect (which is completely analog to the findings in Chen et al. (2000a) on demand forecasting).

## 2.2. Stochastic lead times without forecasting

In Duc et al. (2008), stochastic lead times are investigated under the assumption that they are i.i.d. The simplest two-stage supply chain is analyzed with a first-order autoregressive AR(1) demand process and an extension to a mixed first-order

autoregressive moving-average ARMA(1,1). More precisely, the demands from customers to the retailer constitute the first-order autoregressive moving-average AR(1), that is,  $\{D_t\}_{t=-\infty}^{\infty}$  is a stationary sequence of random variables which satisfy:

$$D_t = \mu + \rho D_{t-1} + \epsilon_t,$$

where  $\mu > 0$ ,  $|\rho| < 1$  and  $\{\epsilon_t\}_{t=-\infty}^{\infty}$  is a sequence of independent identically distributed random variables such that  $\mathbb{E}\epsilon_t = 0$  and  $\text{Var}\epsilon_t = \sigma^2$ . It is easy to notice that  $\mathbb{E}D_t = \mu_D = \frac{\mu}{1-\rho}$ ,  $\mathbb{E}D_t = \mu_D = \frac{\mu}{1-\rho}$ ,  $\text{Var}D_t = \sigma_D^2 = \frac{\sigma^2}{1-\rho^2}$  and the correlation coefficient  $\text{Corr}(D_t, D_{t+1}) = \rho$ . Moreover, it is assumed that the demands are forecast using the minimum-mean-squared-error forecasting method. If  $\hat{D}_{t+i}$  denoted the forecast for a demand for the period  $t + i$  at the beginning of a period  $t$  (i.e. after  $i$  periods), then employing the minimum-mean-squared-error forecasting method we get:

$$\begin{aligned} \hat{D}_{t+i} &= \mathbb{E}(D_{t+i} | D_{t-1}, D_{t-2}, \dots) \\ &= \mu_D(1 - \rho^{i+1}) + \rho^{i+1}D_{t-1}, \end{aligned} \quad (8)$$

where  $D_{t-j}$   $j = 1, 2, \dots$  are demands which have been observed by the retailer until the beginning of a period  $t$ . Then, the lead time demand at the beginning of the period  $t$  is defined by Duc et al. (2008) as follows.

$$\hat{D}_t^L = \sum_{i=0}^{L_t-1} \hat{D}_{t+i},$$

where  $\hat{D}_{t+i}$  is given in Eq. (8). Let us notice that the above lead time demand forecast is not practically feasible, because we do not know the value of  $L_t$  at the beginning of the period  $t$ . Practically, to place an order, we must forecast demands and lead times which means that in the above lead time demand forecast we need to substitute a lead time prediction  $\hat{L}_t$  instead of  $L_t$ . As in the previous model, the retailer uses the order-up-to-level policy and the level of inventory  $S_t$  is given in (2). The variance of the forecast error for the lead time demand and the order quantity  $q_t$  placed by the retailer at the beginning of a period  $t$  are defined in (3) and (4), respectively. The main result of Duc et al. (2008) is the following.

**Theorem 2** *Under the above assumptions with the minimum-mean-squared-error forecasting method the bullwhip effect measure is:*

$$BM = \frac{\text{Var}q_t}{\text{Var}D_t}$$

Duc et al. (2008) give numerical examples and calculate the value of  $BM$  for specific distributions of  $L_t$ , for example, three-point distribution, geometric distribution, Poisson distribution, and discrete uniform distribution. The plots of  $BM$  as a function of the autoregressive coefficient  $\rho$  for a fixed  $\sigma_D/\mu_D$  are presented. It is interesting to note that the minimal value of  $BM$  is attained for  $\rho$  around  $-0.6$  and  $-0.7$ . The maximal value of  $BM$  is for  $\rho$  around  $0.6$  or  $1$ .

In Duc et al. (2008), the results are extended for ARMA(1,1) demand processes (the mixed first-order autoregressive moving-average process). In this case, the structure of demands is defined as follows:

$$D_t = \mu + \rho D_{t-1} + \epsilon_t - \theta \epsilon_{t-1}.$$

where  $\mu$ ,  $\rho$ , and  $\epsilon_t$  are the same as in the case of AR(1) demand process and  $|\theta| < 1$ . Then, under the same assumptions (the order-up-to-level inventory policy and the minimum-mean-squared-error forecasting method), the bullwhip effect measure is given (see Duc et al., 2008).

**Theorem 3** *Under ARMA(1,1) demand process with the minimum-mean-squared-error forecasting method the bullwhip effect measure is:*

$$\begin{aligned} BM &= \frac{\text{Var}q_t}{\text{Var}D_t} \\ &= \frac{(1 - \rho^2)(1 - \theta)[1 - \theta + 2(\theta - \rho)\mathbb{E}\rho^{L_t}] + 2(\rho - \theta)^2[\mathbb{E}\rho^{2L_t} - \rho(\mathbb{E}\rho^{L_t})^2]}{(1 - \rho)^2 + (1 + \theta^2 - 2\theta)} + \frac{2\mu_D^2\sigma_L^2}{\sigma_D^2} \end{aligned}$$

Numerical results for the case of ARMA(1,1) demand process provide the same trends as those of the AR(1) case.

### 2.3. Stochastic lead times with forecasting

In the work of Michna and Nielsen (2013), the impact of lead time forecasting on the bullwhip effect is investigated. It is assumed that lead times and demands are forecast separately which seems to be a very natural and practical approach if they are mutually independent. More precisely, the lead time demand prediction is the following:

$$\hat{D}_t^L = \hat{L}_t \hat{D}_t = \frac{1}{mn} \sum_{i=1}^m L_{t-i} \sum_{i=1}^n D_{t-i}, \quad (9)$$

where we use the moving-average method for lead times and demands with the delay parameters  $m$  and  $n$ , respectively. Moreover, we assume that lead times and demands constitute i.i.d. sequences which are mutually independent. The motivation for the lead time demand forecasting given in Eq. (9) follows  $\mathbb{E}D_t^L = \mathbb{E}L\mathbb{E}D$  (see eq. (1)) (under the assumptions that demands and lead times are mutually independent), and employing natural estimators of  $\mathbb{E}L$  and  $\mathbb{E}D$ , we arrive at Eq. (9). Under the same assumptions as in the previous models on the policy and the lead time demand forecast error, the following result is proven (Michna & Nielsen, 2013).

**Theorem 4** *The measure of the bullwhip effect in a two-stage supply chain has the following form:*



$$BM = \frac{Varq_t}{VarD_t} = \frac{2\sigma_L^2(m+n-1)}{m^2n^2} + \frac{2\mu_D^2\sigma_L^2}{\sigma_D^2m^2} + \frac{2\mu_L^2}{n^2} + \frac{2\mu_L}{n} + 1.$$

The above theoretical model shows that one cannot avoid lead time forecasting when placing orders and the variance of orders will increase dramatically if a crude estimation of lead time (e.g. small  $m$ ) or no estimation is used (e.g. assuming a constant lead time when placing orders). Moreover, demand-signal processing and lead time signal processing, that is, the practice of adjusting demand and lead time forecasts which result in adjusting the parameters of the inventory replenishment rules are the main and equally important causes of the bullwhip effect.

To confirm the theoretical results derived in Michna and Nielsen (2013) and show the impact of stochastic lead times and their forecasting on the bullwhip effect, we simulate the bullwhip effect measure in a supply chain which consists of three echelons. This supply chain is shown in Figure 1.

We conducted two experiments. In the first, we put constant customer demands and stochastic lead times to investigate the influence of stochastic lead times and their forecasting on their own that is we exclude the impact of stochastic demands at the customer level and their forecasting. This simulation confirms the importance of the lead time signal processing. In the second experiment, we include stochastic demands and their forecasting and can confirm the theoretical conclusion derived in Michna and Nielsen (2013). Both experiments are conducted in Matlab and a Monte Carlo method is used to get variance of orders at every stage of the supply chain. Moreover, we take realistic parameters of the underlying supply chain. Thus, going into details in the first simulation, we assume that client demands are deterministic, that is, during a given period (this will be one time unit, e.g. a day), we observe the same constant demand  $D$ . Above the customers in our supply chain, we have a retailer, a manufacturer, and a supplier. Between the manufacturer and the retailer, there are stochastic lead times which create an i.i.d. sequence (i.e. they are the delivery times of the manufacturer to the retailer). Similarly, we observe random lead times between the supplier and the

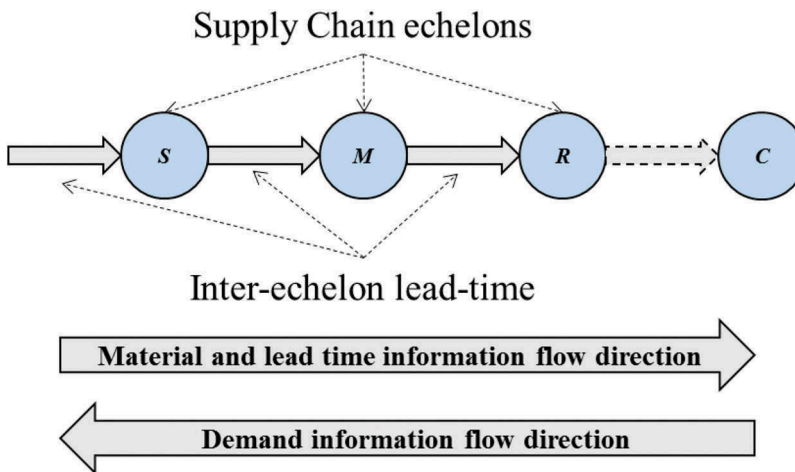


Figure 1. Simulated supply chain setup.

**Table 1.** The bullwhip effect measure for stochastic lead times and constant customer demands – manufacture/retailer orders.

$m/n$	1	2	6	10	20
1	61.6592	17.9880	4.4984	3.3532	2.4048
3	39.1578	14.5778	4.3510	3.1641	2.4692
6	44.2991	14.4909	5.4784	3.0812	2.4720
10	42.9382	13.8638	4.3274	3.6823	2.4074
15	42.4075	14.5218	4.0920	3.1734	2.5155
20	43.292	14.194	4.150	3.165	2.744

manufacturer, and they constitute an i.i.d. sequence. These two sequences are mutually independent. Moreover, we take the review period equal to one time unit (a day), and the lead times are discrete uniform random variables taking on values 1, 2, ..., 7 (time units, e.g. days). The retailer uses the order-up-to-level policy and the moving-average method to predict lead times with the delay parameter  $m$  (customer demands are a constant equal to 5000 – e.g. a number of units of goods, so they are not predicted by the retailer). Similarly, the manufacturer places orders to its supplier, that is, he uses the order-up-to-level policy and the moving-average method to predict lead times with the delay parameter  $m$  and the demands with the delay parameter  $n$  (the demands of the retailer are now random by random lead times in his lead time demand forecast). In Table 1, the simulation results are given for the ratio of variances of the manufacturer and the retailer orders (variance of customer demand is zero). The simulation results show that the bullwhip effect still exists and is quite big even if customer demands are constant but lead times between the echelons are random. Here, one can conclude that the lead time signal processing alone is a fundamental cause of the bullwhip effect just like the demand-signal processing. Moreover, we get a very interesting feature of this supply chain (i.e. with deterministic customer demand) that the delay parameter of demand forecasting  $n$  smoothes bullwhip much faster than the delay parameter of lead time forecasting  $m$ .

Under the same assumptions as above, we simulate the bullwhip effect adding that customer demands are stochastic and i.i.d. following a uniform distribution with the range (4500; 5500) the same number of unit goods as before and independent of lead times. Let us notice that the mean value of demands equals the deterministic demand in the first experiment that is 5000 units of goods. In Table 2, the bullwhip effect at the retailer stage is given, that is, the quotient of the retailer variance and the customer demand variance. Table 3 shows the same as in Table 2 but calculated theoretically using the formula of Th. 4. Here, we get the reverse behavior from the case of deterministic demands. That is, the delay parameter of lead time forecasting  $m$  smoothes bullwhip much faster than the delay parameter of demand forecasting  $n$ .

**Table 2.** The bullwhip effect measure at the retailer stage for stochastic lead times and stochastic customer demands.

$m/n$	1	2	6	10	20
1	2506.6	2336.7	2392.4	2380.9	2549.4
3	310.13	280.31	269.90	267.19	265.30
6	107.60	78.70	68.65	71.21	68.86
10	67.313	37.270	26.465	25.997	25.889
20	46.8218	19.3528	9.2602	8.1362	7.4563

**Table 3.** The bullwhip effect measure at the retailer stage for stochastic lead times and stochastic customer demands calculated theoretically.

$m/n$	1	2	6	10	20
1	2449.0	2417.0	2404.6	2402.9	2401.9
3	310.33	280.55	270.08	268.89	268.19
6	109.00	80.055	69.956	68.820	68.160
10	65.800	37.220	27.255	26.135	25.485
20	47.400	19.105	9.2361	8.1258	7.4820

**Table 4.** The bullwhip effect measure at the manufacturer stage for stochastic lead times and stochastic customer demands.

$m/n$	1	2	6	10	20
1	194,700	41,742	10,720	8024.0	5891.9
3	13,495	4221.1	1191.6	856.245	676.579
6	5821.1	1216.1	364.729	226.268	171.459
10	3671.5	581.341	112.412	93.529	62.333
20	2840.2	316.308	37.244	23.710	18.972

(see Tables 2 and 3). Moreover, we get that the theoretical result of Michna and Nielsen (2013) (Th. 4) and our simulation coincide perfectly. Finally, in Table 4, we have the bullwhip effect measure at the manufacturer stage, that is, the ratio of the manufacturer order variance and the customer demand variance (we could count the quotient of the manufacturer order variance and the retailer order variance, but it is easy to get this having also the ratio of the retailer variance and the customer demand variance [see Tables 2 and 3]). The simulation results for the bullwhip effect at the manufacturer stage show that the delay parameter of demand forecasting  $n$  and the delay parameter of lead time forecasting  $m$  dampen the effect with a similar strength.

### 3. Conclusions and future research opportunities

The main conclusion from our research is that stochastic lead times boost the bullwhip effect. More precisely, we deduce from the presented models, that the increase of the expected value and variance of lead times and the probability of the longest lead time amplify the bullwhip effect. Moreover, the delay parameter of the prediction of demands, the delay parameter of the prediction of lead times, and the delay parameter of the prediction of lead time demands are, depending on the model, crucial parameters which can dampen the bullwhip effect. The simulations yield a very interesting conclusion that the strength of dampening the bullwhip effect is different for the demand prediction and lead time prediction depending on the stage of a supply chain. We must also note that in all the presented models, the bullwhip effect measure contains the term  $\frac{2\mu_D^2\sigma_L^2}{\sigma_D^2}$  (see Th. 1, 2, 3, and 4) and, except for the model of Duc et al. (2008), this term can be terminated by the prediction (going with  $n$  or  $m$  to  $\infty$ ). The conclusions especially the impact of lead time signal processing on the bullwhip effect can be applied in supply chains where stochastic lead times are observed and the moving-average forecasting method is used which are quite common at many manufacturers. Theoretical formulas and simulations reveal parameters which are essential to dampen the bullwhip effect, e.g. the number of the past observation used to predict future

demand and future lead time or the parameters of the demand distribution and the lead time distribution.

Future research on quantifying the bullwhip effect should aim at supply chains which observe stochastic lead times with a different structure than i.i.d. and dependence between lead times and demands. To improve the modeling of lead times requires data such as can be gathered, for example, by technologies like radio frequency identification (Nielsen, Lim, & Nielsen, 2010). Another challenge in bullwhip modeling is the problem of lead time forecasting and its impact on the bullwhip effect. A member of a supply chain placing an order must forecast lead times to determine an appropriate inventory level to fulfill its customer orders in a timely manner. This implies that lead times influence orders. In turn, orders can impact lead times. This feedback loop can be the most important factor causing the bullwhip effect and seems to be relevant but very difficult to quantify. The next problem arising in the presence of stochastic lead times is crossover of lead times and its impact on bullwhip (see Wang & Disney, 2017) which is now a very painful phenomenon of modern supply chains. Moreover, simultaneous dampening of bullwhip and inventory level variability can significantly decrease costs which yields that order variance and net stock variance should be analyzed (see Wang & Disney, 2017). Other replenishment policies such as proportional order-up-to-level policy should be investigated in the context of bullwhip and stock level variability when stochastic lead times are assumed with crossovers. Thus, the spectrum of models which reflect structures of many realistic supply chains and must be investigated is very wide. However, these problems do not seem to be solved easily by providing analytical models alone (see Wang & Disney, 2017) for the progress and trends in the bullwhip effect research).

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